

**MATHEMATICS  
HIGHER LEVEL  
PAPER 1**

Candidate number

--	--	--	--	--	--	--	--

Thursday 6 May 2004 (afternoon)

2 hours

---

**INSTRUCTIONS TO CANDIDATES**

- Write your candidate number in the box above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the appropriate box on your cover sheet  
*e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. The polynomial  $x^3 - 2x^2 + ax + b$  has a factor  $(x - 1)$  and a remainder 8 when divided by  $(x + 1)$ . Calculate the value of  $a$  and of  $b$ .

*Working:*

*Answer:*

2. Given that  $\frac{dy}{dx} = 2x - \sin x$  and  $y = 2$  when  $x = 0$ , find an expression for  $y$  in terms of  $x$ .

*Working:*

*Answer:*

3. For  $0 \leq x \leq 6$ , find the coordinates of the points of intersection of the curves

$$y = x^2 \cos x \text{ and } x + 2y = 1.$$

*Working:*

*Answer:*

4. A geometric series has a negative common ratio. The sum of the first two terms is 6. The sum to infinity is 8. Find the common ratio and the first term.

*Working:*

*Answer:*

$r =$

$u_1 =$

5. The composite transformation  $T$  is defined by a clockwise rotation of  $45^\circ$  about the origin followed by a reflection in the line  $x + y = 0$ . Calculate the  $2 \times 2$  matrix representing  $T$ .

*Working:*

*Answer:*

6. The weights of adult males of a type of dog may be assumed to be normally distributed with mean 25 kg and standard deviation 3 kg. Given that 30% of the weights lie between 25 kg and  $x$  kg, where  $x > 25$ , find the value of  $x$ .

*Working:*

*Answer:*

7. The point  $P(1, p)$ , where  $p > 0$ , lies on the curve  $2y^2 - x^3y = 15$ .

(a) Calculate the value of  $p$ .

(b) Calculate the gradient of the tangent to the curve at  $P$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

8. Given that  $\mathbf{a} = (i + 2j + k)$ ,  $\mathbf{b} = (i - 3j + 2k)$  and  $\mathbf{c} = (2i + j - 2k)$ , calculate  $(\mathbf{a} - \mathbf{b}) \cdot (\mathbf{b} \times \mathbf{c})$ .

*Working:*

*Answer:*

\_\_\_\_\_

9. The function  $f$  is defined on the domain  $[-1, 0]$  by

$$f : x \mapsto \frac{1}{1+x^2}.$$

- (a) Write down the range of  $f$ .
- (b) Find an expression for  $f^{-1}(x)$ .

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_

10. The line  $x-1 = \frac{y+1}{2} = \frac{z}{3}$  and the plane  $\mathbf{r} \cdot (\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = 1$  intersect at the point P. Find the coordinates of P.

*Working:*

*Answer:*

11. (a) Find  $\int_0^m \frac{dx}{x^2+4}$ , giving your answer in terms of  $m$ .  
(b) Given that  $\int_0^m \frac{dx}{x^2+4} = \frac{1}{3}$ , calculate the value of  $m$ .

*Working:*

*Answers:*

- (a) \_\_\_\_\_  
(b) \_\_\_\_\_

12. Marian shoots ten arrows at a target. Each arrow has probability 0.4 of hitting the target, independently of all other arrows. Let  $X$  denote the number of these arrows hitting the target.

(a) Find the mean and standard deviation of  $X$ .

(b) Find  $P(X \geq 2)$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

13. A desk has three drawers. Drawer 1 contains three gold coins, Drawer 2 contains two gold coins and one silver coin and Drawer 3 contains one gold coin and two silver coins. A drawer is chosen at random and from it a coin is chosen at random.

- (a) Find the probability that the chosen coin is gold.
- (b) Given that the chosen coin is gold, find the probability that Drawer 3 was chosen.

*Working:*

*Answers:*

(a) \_\_\_\_\_

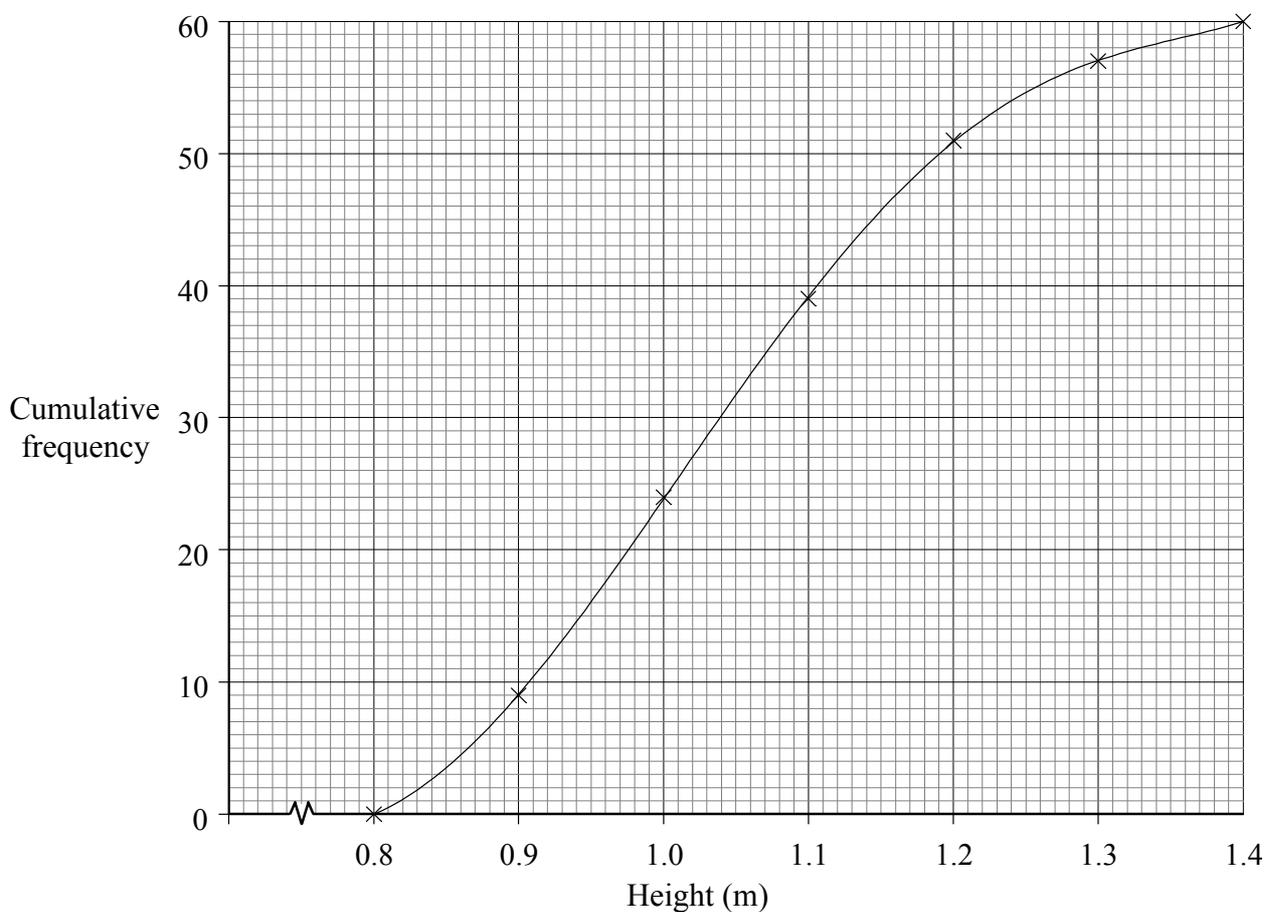
(b) \_\_\_\_\_

14. Find  $\int x^2 e^x dx$ .

*Working:*

*Answer:*

15. The heights of 60 children entering a school were measured. The following cumulative frequency graph illustrates the data obtained.



Estimate

- (a) the median height;
- (b) the mean height.

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_

16. Solve the inequality

$$\left| \frac{x+12}{x-12} \right| \leq 3.$$

*Working:*

*Answer:*

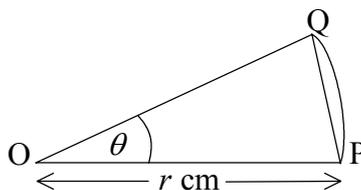
17. The function  $f$  is defined by  $f : x \mapsto 3^{x^2}$ .

Find the solution of the equation  $f'(x) = 2$ .

*Working:*

*Answer:*

18. The figure shows a sector OPQ of a circle of radius  $r$  cm and centre O, where  $\widehat{POQ} = \theta$ .



The value of  $r$  is increasing at the rate of 2 cm per second and the value of  $\theta$  is increasing at the rate of 0.1 rad per second. Find the rate of increase of the area of the sector when  $r = 3$  and  $\theta = \frac{\pi}{4}$ .

*Working:*

*Answer:*

19. Let  $f(x) = x^3 \cos x$ ,  $0 \leq x \leq \frac{\pi}{2}$ .

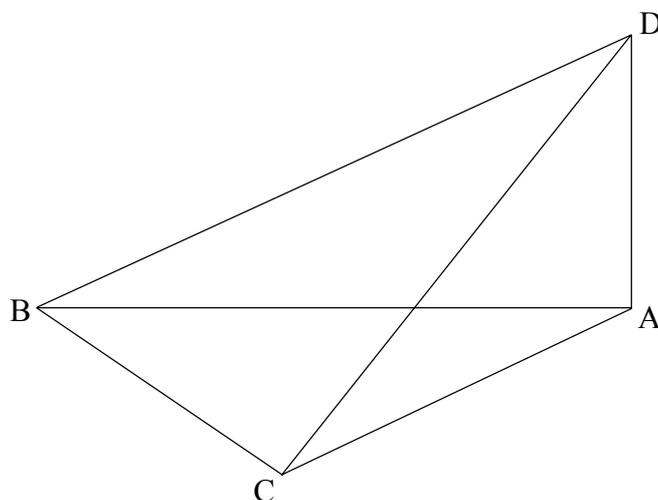
- (a) Find  $f'(x)$ .
- (b) Find the value of  $x$  for which  $f(x)$  is a maximum.
- (c) Find the  $x$ -coordinate of the point of inflexion on the graph of  $f(x)$ .

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_

20. The following three dimensional diagram shows the four points A, B, C and D. A, B and C are in the same horizontal plane, and AD is vertical.  $\hat{A}BC = 45^\circ$ ,  $BC = 50$  m,  $\hat{A}BD = 30^\circ$ ,  $\hat{A}CD = 20^\circ$ .



Using the cosine rule in the triangle ABC, or otherwise, find AD.

*Working:*

*Answer:*